# A SYSTEMATIC METHOD FOR TRACER TEST ANALYSIS:  AN EXAMPLE USING BEOWAWE TRACER DATA

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## ABSTRACT

Quantitative analysis of tracer data using moment analysis requires a strict adherence to a set of rules which include data normalization, correction for thermal decay, deconvolution, extrapolation, and integration.  If done correctly, the method yields specific information on swept pore volume, flow geometry and fluid velocity, and an understanding of the nature of reservoir boundaries.  All calculations required for the interpretation can be done in a spreadsheet.

The steps required for moment analysis are reviewed in this paper.  Data taken from the literature is used in an example calculation.

## INTRODUCTION

Tracer testing remains a crucial tool for characterizing geothermal resources, with more than 100 geothermal tracer tests conducted worldwide in the last 40 years (M. Adams, 2003, personal communication).  A vast majority of these tests were interpreted qualitatively, ignoring the temporal evolution of the tracer breakthrough curve and resulting in gross test interpretation (e.g., size of the arrow indicative of relative tracer flow).  Others have used tracer tests to constrain numerical models, using the data to estimate heat transfer parameters (Robinson and Tester, 1984; Axelsson et al., 2001), or to constrain reservoir-scale numerical models (e.g., Gunderson et al., 2002; Bloomfield and Moore, 2003).

There are a host of tracer test analysis methods that consider the temporal behavior of tracers.  The methods were originally developed for closed reactor vessels (Danckwerts, 1958; Levenspiel, 1972), but have been applied to more general conditions of open boundaries (Pope et al., 1994; Sinha et al., 2004), characterization of fractured media under continuous tracer reinjection (Robinson and Tester, 1984), and estimates of flow geometry (Shook, 2003).  These methods have a rigorous mathematical basis and offer additional information on the subsurface.  The analysis is useful independently, but also can be used to constrain numerical models by defining interwell volume and flow geometry.

The methods and applications mentioned above are all based on analysis of tracer residence times.  The mean residence time, or first temporal moment, is the most useful single property derived from a tracer test, although other properties have been used as well.  Levenspiel (1972) shows the total pore volume swept by a tracer can be determined from its mean residence time.  There are certain restrictions inherent in the calculation; for example, steady state conditions and conservative tracer behavior.  Nevertheless, the method has a rigorous mathematical basis, and has been extensively validated analytically and experimentally.

The purpose of this paper is to describe the individual steps necessary to apply moment analysis to tracer test interpretation.  For the purposes of illustration, the Beowawe tracer dataset described by Rose et al. (2004) is used in an example interpretation.  No attempt to verify the dataset was made, and some assumptions regarding injection were made, so this might be better called a Beowawe-like tracer test.  The dataset serves a useful purpose in that it is actual field data and has been analyzed and published.  A discussion of assumptions and limitations of the method follow the example interpretation.

The discussions below make several assumptions regarding the tracer (fluorescein in this case).  First, we assume it is conservative; that is, it does not adsorb or volatize.  We also assume that, at the concentrations injected, it is an ideal tracer; it does not affect the flow properties of the liquid phase (density, viscosity, etc.).  The latter assumption is almost certainly true, as the injected concentration is approximately 0.4 wt%.  The former assumption, however, is of potential concern.  Fluorescein has shown sorptive behavior under certain conditions (Sabatini and Austin, 1991), and adsorption was

suspected under geothermal conditions (Gunderson et al., 2002). Axelsson et al. (2001) show that fluoroscein was non-sorptive under conditions at the Laugaland geothermal field. However, adsorption results from rock-water interactions and is site-specific.

Nevertheless, for the purposes of this discussion, fluorescein is assumed conservative. On July 13, 1994 (t=0), 91 kg of fluorescein was mixed with 23 $m^3$ of water, and injected as an aqueous slug. Injection and production mass rates are both assumed constant, and equal to $7.88 \times 10^5$ T/hr ($18.9 \times 10^6$ kg/d). This information was extracted from the discussion by Rose et al. (2004). Data was collected over the course of the following 9 years. Individual production rates from the three production wells were not available; rather, an averaged tracer history was reported. The raw concentration history for the Beowawe tracer test was given to the present author by P. Rose. The raw data is plotted in Figure 1.
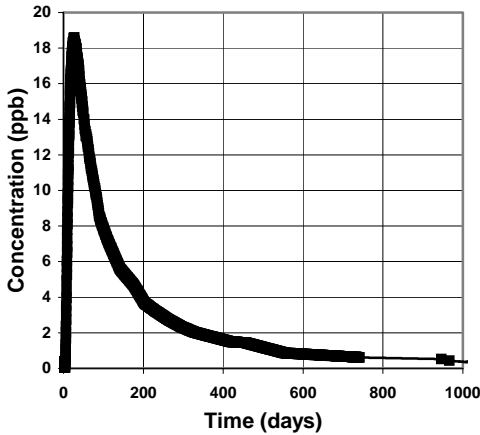


*Figure 1. Tracer history from Beowawe test, as reported by Rose et al. (2004). Symbols are individual observations.*

## NORMALIZING THE TRACER HISTORY

The method of moments is based on age distribution functions as originally described by Danckwerts (1958). To avoid ambiguity in terminology, however, we will use the nomenclature of Levenspiel (1972). The age distribution function is referred to as E(t), and has units of (1/t). We will refer to the tracer concentration history in Figure 1 as C(t), whose units in the present case are (ppb). In order to convert C(t) to E(t), we multiply by the mass flow rate $\rho q$ and divide by the total mass of tracer injected, M:

$$E(t) = \frac{C(t)\rho q}{10^9 M} \qquad (1)$$

Normalizing E(t) in such a fashion has distinct advantages. First, normalization of the output signal accommodates treating the input signal as a Dirac delta function (e.g., a pulse injection). The properties of the Dirac delta function are required to deconvolve the tracer history as discussed in a following section. The area under the curve E(t) vs. t is unity in a closed system (100% tracer recovery), so the normalized curve offers a quick means of evaluating the flow system. The normalization also puts all tracer response curves on an equal footing, making direct comparison easier. It is conventional to normalize time as well, so E is independent of formation size, flow rates, etc. However, time is usually normalized by the mean residence time (Levenspiel, 1972), the determination of which is one purpose of this paper.

Normally a tracer history must first be corrected for thermal degradation using the Arrhenius equation (Levenspiel, 1972). However, Rose et al. (2004) indicate that fluoroscein is not expected to degrade at the temperatures encountered in the tracer test, so no correction was applied to the example tracer data.

## DECONVOLVING THE TRACER HISTORY

When tracer is reinjected, the observed tracer history is a combined response to the initial slug tracer injection and the continuous recycling of the produced tracer. Moment analysis is based on the response to slug tracer injection, so we must first remove the effect of tracer recycling before calculating residence times and swept volumes. The convolution integral is used to deconvolve the tracer response (Levenspiel, 1972):

$$E_{app}(t) = \int_0^t E_{in}(t-\tau)E(\tau)d\tau \qquad (2)$$

Equation (2) states the observed (apparent) residence time distribution, $E_{app}(t)$, is a result of injection $E_{in}$ and the true residence time distribution, E(t). Following arguments presented by Robinson and Tester (1984):

$$E_{app}(t) = \frac{C(t)\rho q}{M} \qquad (3a)$$

$$E_{in} = \delta(t) + \frac{1}{1-f_{loss}} \cdot \frac{C(t)\rho q}{M} \qquad (3b)$$

Substituting Equations 3 in Equation 2 gives

$$E_{app}(t) = \int_0^t \left\{ \delta(t) + \frac{1}{1-f_{loss}} \cdot E(t-\tau) \right\} E(\tau)d\tau \qquad (4)$$

Using the definition of the delta function and rearranging Equation 4 gives the correction needed to remove the effects of reinjection:

$$E(t) = E_{app}(t) - \frac{1}{1 - f_{loss}} \int_0^t E(t - \tau)E(\tau)d\tau \qquad (5)$$

The integral in Equation 5 must be calculated anew at each time using the current tracer concentration, C(t), the previous injection history, C(t-τ), and residence time ages, E(τ). At the upper limit of the integration, the argument is zero, so the current residence time age, E(t), can be calculated explicitly at each time step.

The raw $E_{app}(t)$ data and the deconvolved E(t) data for the example are given in Figure 2 below. We have assumed that $f_{loss}$ is zero in this example. That is, the concentration that is produced is the same as that subsequently injected. This would be the case even if water were lost in cooling towers, for example, if makeup water were used to maintain a constant injection rate. Note that the deconvolved signal is always a subset of the raw data.
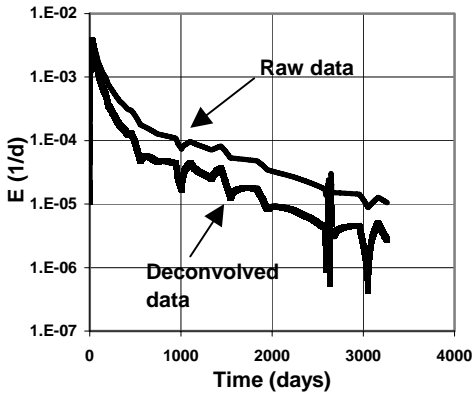


*Figure 2. Raw vs. Deconvolved Age Distribution Functions, E(t). Note the deconvolved signal is always a subset of the raw data.*

## CALCULATING MEAN RESIDENCE TIMES

The mean residence time, or first temporal moment, of a tracer is determined directly from the deconvolved, thermally-corrected age distribution function, E(t) by the following equation:

$$t^* = \frac{\int_0^\infty E(t)tdt}{\int_0^\infty E(t)dt} - \frac{t_s}{2} \qquad (6)$$

If the data is known only at discrete times, $t_i$, the integral can be approximated as:

$$t^* = \frac{\sum_{i=1}^N E_i t_i \Delta t_i}{\sum_{i=1}^N E_i \Delta t_i} - \frac{t_s}{2}$$

If the data itself is sufficiently smooth, a function may be fit to the tracer history, and Equation 6 may be calculated analytically. Usually, however, the data is integrated numerically in a spreadsheet. The trapazoid rule can be used to determine t* accurately.

## EXTRAPOLATING THE HISTORY TO LONG TIMES

Sampling for tracer is frequently terminated long before the tracer concentration is zero. Because the first moment is a time-weighted average, failure to include late time data leads to underprediction of mean residence time, and consequently, in pore volume calculations. This is fixed by breaking the integrals in Equation 6 in two:

$$t^* = \frac{\int_0^{t_b} E(t)tdt + \int_{t_b}^\infty E(t)tdt}{\int_0^{t_b} E(t)dt + \int_{t_b}^\infty E(t)dt} - \frac{t_s}{2} \qquad (7)$$

If a curve is fit through the late time tracer data, the second integral in numerator and denominator can be evaluated in closed form. For example, if the plot of log(E) vs. time is linear for $t > t_b$, the decline is exponential and the tracer data can be represented as:

$$E(t) = be^{-at} \quad \text{for } t > t_b \qquad (8)$$

Exponential decline is probably the most common tracer decline observed. Other curves may be used, but only with caution. Table 1 below shows the potential dangers of other types of curves used to extrapolate the example data. Both linear and power law extrapolation give non-physical results for this example, and therefore should not be used.

## DETERMINING PORE VOLUME

Pore volume estimates follow directly from the mean residence time (Levenspiel, 1972). For open boundaries, multiple production wells, and/or incomplete recovery of injected tracer, the pore volume swept by the tracer is given as (Pope et al., 1994)

$$V_p = \frac{m}{M} q_{inj} t^* \tag{9}$$

The fractional recovery of tracer at any given well, m/M, is the integral of E(t) for that well; again, a simple spreadsheet calculation.

*Table 1. Summary of models used for extrapolating tracer data.*

| Form | Equation | Comments |
|---|---|---|
| Linear (Rose et al. 2004) | $E = 1.42 \times 10^{-4} - 4.74 \times 10^{-8} t$ **Predicts C < 0 for t > 2900 days** | Non-physical |
| Exponential | $E = 2.17 \times 10^{-4} e^{-0.0015t}$ $R^2 = 0.914$ | |
| Power law | $E = 11407 t^{-1.74}$ $R^2 = 0.878$ **Too slow a decline; E > 0 at 30 years.** | Not feasible |

## CALCULATING FLOW GEOMETRY

Shook (2003) showed that the flow and pore volume geometry of a formation (fractured or otherwise) could be estimated directly from a tracer test. Individual flow paths are imagined as streamlines, independent of the exact formation properties. The flow capacity, $f_i$, of the individual streamline is its specific velocity, relative to the bulk velocity. The storage capacity, $c_i$, is the pore volume associated with that streamline. We can approximate the true streamline geometries, F and C, from the E(t) curve as

$$C(t) \cong \frac{\int_0^t E(\tau)\tau d\tau}{\int_0^\infty E(t)t dt} \tag{10a}$$

$$F(t) \cong \frac{\int_0^t E(\tau)d\tau}{\int_0^\infty E(t)dt} \tag{10b}$$

From the equations given above, it is clear that C(t) and F(t) are incremental summations of streamline geometries. A value $F_n$, for example, is the fraction of streamlines with velocity equal to or greater than streamline n. $C_n$ is the fraction of the total pore volume comprising those streamlines. Flow, F, and storage, C, capacity are most often plotted on a F-C plot. The shape of the F-C curve is useful as a diagnostic tool; for example, indicating what fraction

of the pore volume contributes what fraction of the fluid flow. The slope of the F-C curve is the instantaneous fluid velocity (Lake, 1989, p 195), which can be useful in predicting thermal velocities arising from injection. This application remains a goal of the INEEL tracer test interpretation program.

## EXAMPLE MOMENT ANALYSIS

The methods discussed above were applied to the "Beowawe-like" tracer test data. The data were first normalized and deconvolved as discussed above. The corrected curve in Figure 2 is that used in these example calculations. Late time data was fit with exponential curve. The data from t > 1630 days shows good correlation between the data and the curve. The exponential equation is given in Table 1.

The first moment is calculated thusly. First, the trapazoid rule can be applied to the corrected E(t) data to calculate the integrals in Equation 7 from 0 < t < 1630 days. Symbolic integration of the exponential decline equation shows

$$\int_{t_b}^\infty E(t)t dt = \frac{b}{a^2} e^{-at_b}(1 + at_b) \tag{11}$$

and

$$\int_{t_b}^\infty E(t)dt = \frac{b}{a} e^{-at_b} \tag{12}$$

Using the values of a and b from Table 1, numerator and denominator for the moment calculation can be determined. Results of the calculation are given in Table 2. The mean residence time, t* is 268.8 days. Pore volume swept by the tracer is calculated as

$$V_p = qt^* \frac{m}{M} = (19000 \frac{m^3}{d})(268.8d)(0.396)$$
$$= 2.04 \times 10^6 \text{ m}^3.$$

*Table 2. Summary of piecewise integration of example tracer data.*

| Numerator | Denominator | t* |
|---|---|---|
| $\sum_{i=2}^{1630} \frac{(E_i t_i + E_{i-1}t_{i-1})}{2}\Delta t_i$ $+ \int_{1630}^\infty C t dt$ | $\sum_{i=2}^{1630} \frac{(E_i + E_{i-1})}{2}\Delta t_i$ $+ \int_{1630}^\infty C dt$ | |
| =77.81+28.7 = 106.55 | = 0.384+0.0125 = 0.3965 | 268.8 days |

As an aside, consider the effect of not extrapolating the tracer data to infinite time. By ignoring all data for t > 1500 days, the apparent mean residence time can be calculated as 195.3 days. Thus, failure to include late time data results in an error

(underprediction) in residence time and pore volume estimates of 40%.

The F-C data is also calculated readily from the integrated E(t) data. The F-C curve is shown in Figure 3 for the example data. The F-C curve is useful for conceptualizing the nature of the flow paths. For example, it appears that about 60% of the flow is coming from some 12% of the total pore volume. This indicates a few high permeability streaks (fractures) are dominating the interwell flow.
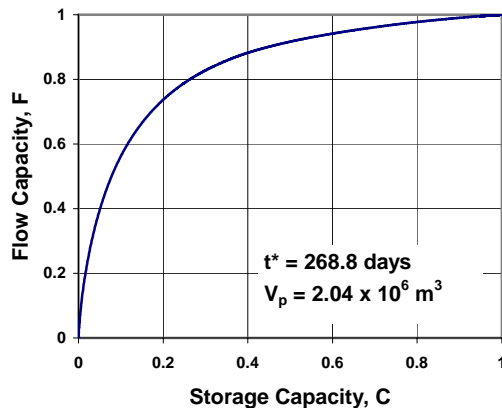


*Figure 3. F-C Diagram for Example*

Other information is available from a review of the tracer data. The tracer history continues to decline, despite continued tracer reinjection. This clearly suggests the Beowawe reservoir is open. That is supported by tracer recovery data as well; only 40% of the tracer injected was recovered. The implication of low recovery is that *for existing flow conditions* (e.g., direction) there is net discharge from the reservoir between points of injection and extraction. It would be interesting to reverse the flow field (exchange injection for production) and see what difference that would have on tracer recovery. There may be a means of optimizing injection/production locations such that net outflow is minimized.

## LIMITATIONS OF THE METHOD

Moment analysis suffers from very few assumptions and limitations. The flow field is required to be in steady state, otherwise the streamlines (and therefore swept volumes) are changing with time. Small excursions from steady state flow can be mitigated in the following manner. Time is replaced as the independent variable with "volumes injected" (qt) in the definitions for E(t) etc. above. Variations in flow up to 15% over time have been treated successfully in this manner. Minor disruptions in flow (e.g., temporary suspension of injection or extraction) can usually be ignored.

Moment analysis is also based on the behavior of the tracer itself. The tracer is just a proxy for the reservoir fluids, so the tracer must behave like the reservoir fluids. Strictly speaking, that implies the tracer be conservative and ideal. The first requirement means the tracer strongly partitions into a single phase, the volume of which we are interrogating. The second requirement implies the tracer does not change the flow field itself. Unfortunately, failure to confirm the tracer requirements above does not preclude a test analysis. The problem is that, in the absence of site-specific verification, we cannot know if the analysis is correct.

A final observation on moment analysis is that it provides information on the pore volume swept by injectate. This is hardly a surprise, since pore volume not contacted by injectate cannot be interrogated via tracers – or any other injection-derived test. There is not, to this author's knowledge, any method that estimates total (swept and unswept) volumes in an open system. All methods proposed suffer from one or more weaknesses, including physical inconsistencies and issues of non-uniqueness.

## SUMMARY

Moment analysis offers a mathematically rigorous means of analyzing the temporal behavior of fluid flow to derive reservoir properties. Developed originally in the late 1950's for closed systems, the methods have been enhanced, and new estimation methods have been developed. The formula for applying moment analysis is straightforward: convert raw data to age distribution functions, correct for thermal degradation, deconvolve the data, extrapolate the tracer tail, and numerically integrate the results. Swept pore volume, degree of outflow, and flow geometry are readily extracted from the analysis. All the calculations can be done in a spreadsheet. The spreadsheet used in the current example can be made available to interested parties.

## ACKNOWLEDGEMENTS

## NOMENCLATURE

a      exponent in the exponential decline equation
        [=] day$^{-1}$

b      cofficient in the exponential decline equation [=] day$^{-1}$

$C(t)$      produced tracer concentration [=] parts per billion (ppb)

$E(t)$      age distribution function of a tracer [=] day$^{-1}$

$E_{in}$      age distribution function injectate [=] day$^{-1}$

$E_{app}$      apparent age distribution function of a tracer – equal to true $E(t)$ if no recycling of tracer occurs [=] day$^{-1}$

$f_{loss}$      fluid loss between extraction and injection that leads to increased injection concentrations. Required in deconvolution calculations.

m      mass of tracer recovered [=] kg

M      mass of tracer injected [=] kg

t      time [=] day

$t_b$      time at which tracer begins exponential decline [=] day

$t_s$      tracer slug injection duration [=] day

$t^*$      first temporal moment, or mean residence time [=] day

$V_p$      pore volume swept by tracer [=] m$^3$

$q_{inj}$      volumetric injection rate [=] m$^3$/day

$\delta$      Dirac delta function

$\rho$      fluid density [=] kg/m$^3$

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